ALTERNATE METHOD OF ANALYSIS

1. <u>Definition of Factor of Safety</u>. This sliding stability criteria is based upon presently acceptable geotechnical principles with respect to shearing resistance of soils and rock, and applies the factor of safety to the least known conditions affecting sliding stability; that is, the material strength parameters. The factor of safety is related to the required shear stress and available shear strength according to Equation 1A:

$$\tau = \frac{\tau_a}{FS} \qquad (1A)$$

where

τ = the required shear stress for safe stability
τ_a = the available shear strength
FS = the factor of safety

The most accepted criteria for defining the available shear strength (τ_a) of a given material is the Mohr-Coulomb failure criteria. Equation 1A may be rewritten as:

$$\tau = (c + \sigma \tan\phi)/FS \qquad (2A)$$

in which

c = the cohesion intercept
σ = the normal stress on the shear plane
ϕ = the angle of internal friction

The ratio $\dfrac{\tau_a}{FS}$ can be considered as the degree of shear mobilization.

2. <u>Solutions for Factor of Safety</u>. The following equations for evaluating sliding stability were developed from the definition of FS and the assumption discussed in paragraph one above. The equations provide FS solutions for both single and multiple-plane failure surfaces, using any number of blocks or wedges.

Inclosure 3

a. Notation:

   c = the cohesion intercept

   U = the uplift force acting under a wedge on the critical potential
       failure plane = uplift pressure x area of critical potential
       failure plane

   A = the area of the critical potential failure plane

   V = all applied vertical forces (body and surcharge) acting on
       an individual wedge

   H = all applied horizontal forces acting on an individual wedge

   α = the angle between the inclined plane of the critical potential
       failure surface and the horizontal ($\alpha > 0$ for upslope sliding;
       $\alpha < 0$ for downslope sliding)

   ϕ = the angle of internal friction along the critical potential
       failure plane considered

   i = the subscript associated with planar segments along the
       critical potential failure surface

   N = the number of wedges in the failure mechanism or number of
       planes making up the critical potential failure surface

   b. Case 1: Single-Plane Failure Surface. Figure 3-1 shows a graphical representation of a single-plane failure mode. Here the critical potential failure surface is defined by a single plane at the interface between the structure and foundation material with no embedment. Equation 3A provides a direct solution for FS for inclined failure planes.

$$FS = \frac{cA + (V \cos \alpha - U + H \sin \alpha) \tan \phi}{H \cos \alpha - V \sin \alpha} \qquad (3A)$$

For the case where the critical potential failure surface can be defined as a horizontal plane ($\alpha = 0$), Equation 3A reduces to Equation 4A:

$$FS = \frac{cA + (V - U) \tan \phi}{H} \qquad (4A)$$

   c. Case 2: Multiple-Plane Failure Surface. This general case is applicable to situations where the structure is embedded and/or where the critical potential failure surface is defined by two or more weak planes. The solution for FS is obtained from Equation 5A:

$$FS = \cfrac{\displaystyle\sum_{i=1}^{N} \cfrac{c_i A_i \cos \alpha_i + (V_i - U_i \cos \alpha_i) \tan \phi_i}{n_{\alpha i}}}{\displaystyle\sum_{i=1}^{N} (H_i - V_i \tan \alpha_i)} \qquad (5A)$$

where

$$n_{\alpha_i} = \cfrac{1 - \cfrac{\tan \phi_i \tan \alpha_i}{FS}}{1 + \tan^2 \alpha_i}$$

Figure 3-2 shows a graphical representation of a multiple (in its simplest form, two planes) plane failure mode.

3. Use of Equations and Limitations of Analytic Techniques.

a. Case One: Single-Plane Failure Surface. The solution for the factor of safety is explicit by use of Equations 3A and 4A. These equations satisfy both vertical and horizontal static equilibrium. However, the user should be aware that in cases for which $\alpha > 0$ (upslope sliding) and where $H/V \leqslant \tan \alpha$, Equation 3A results in a FS = ∞ or a negative FS; in these cases, solutions for FS do not have meaning.

b. Case Two: Multiple-Plane Failure Surface.

(1) Equation 5A is implicit in FS (except when $\phi = 0$ or $\alpha = 0$) since n_α is a function of FS. Therefore, the mathematical solution of Equation 5A requires an iteration procedure. The iteration procedure requires that an initial estimate of FS be inserted into the n_α term and a FS calculated. The calculated FS is then inserted into the n_α term and the process is repeated until the calculated FS converges with the inserted FS. Generally, convergence occurs within four to five iterations. The iteration process can be performed manually or the equation can be easily programmed for a programmable calculator. To facilitate hand solution, a plot of n_α versus α for values of $\tan \phi$/FS is given in Figure 3-3.

(2) Equation 5A is similar to the generalized method of slices for sliding stability criteria. However, in order to develop a simple analytic technique suitable for routine use, the vertical side forces due to impending

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motion of the wedges between slices were assumed to be zero. Therefore, although the equation satisfies complete horizontal static equilibrium, complete vertical equilibrium is in general not satisfied. The FS computed from Equation 5A will be slightly lower than the FS computed from the more complicated techniques which completely satisfy both vertical and horizontal static equilibrium.

(3) The user should be aware that Equation 5A will yield identical solutions for FS with the methods described in the main body of this ETL. The governing wedge equation (equation seven), together with the boundary conditions (equations three and four) to have the system of wedges act as an integral failure mechanism, is mathematically equivalent to Equation 5A. The user may find the more convenient method to be a function of the design situation. Since solutions for FS by these two methods of analysis are identical, and since the mathematical approach is quite different, one can effectively be used as a check on the other.

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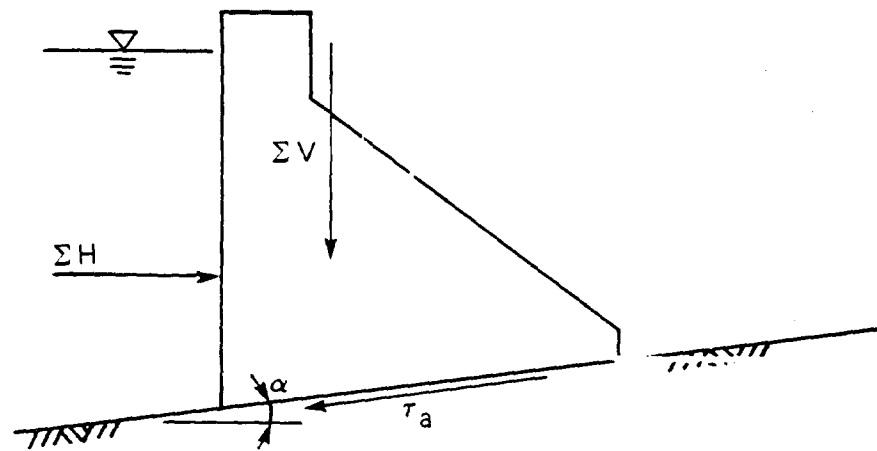
a. Upslope Sliding, $\alpha > 0$
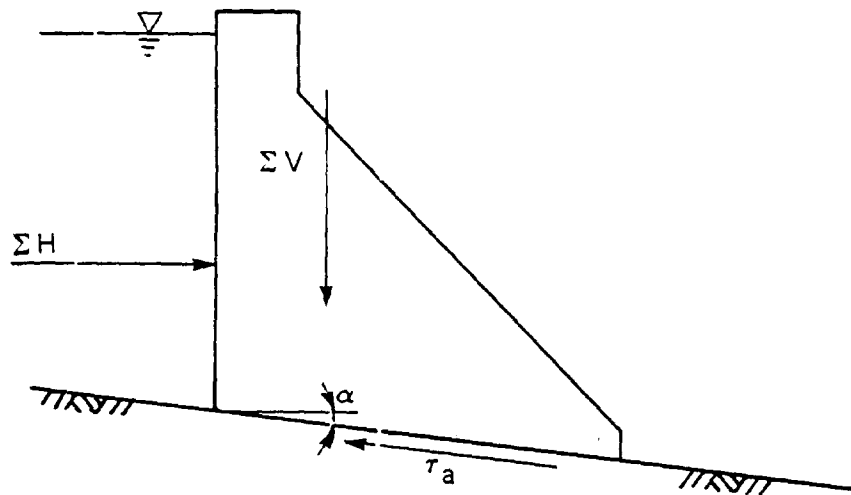
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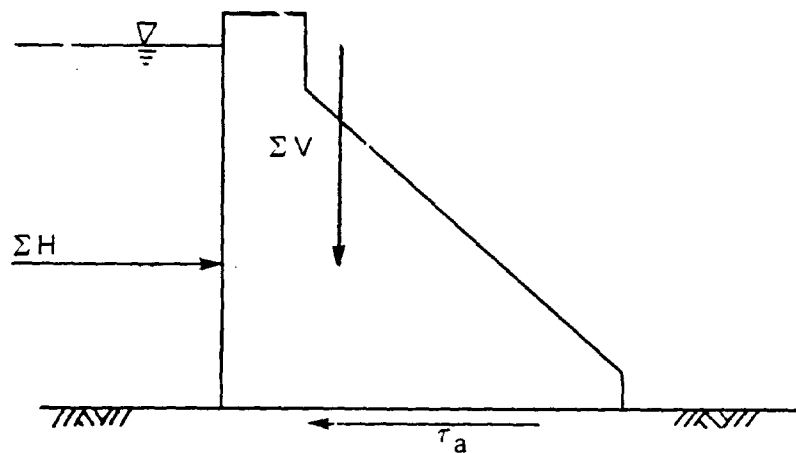
c. Horizontal Sliding, $\alpha = 0$
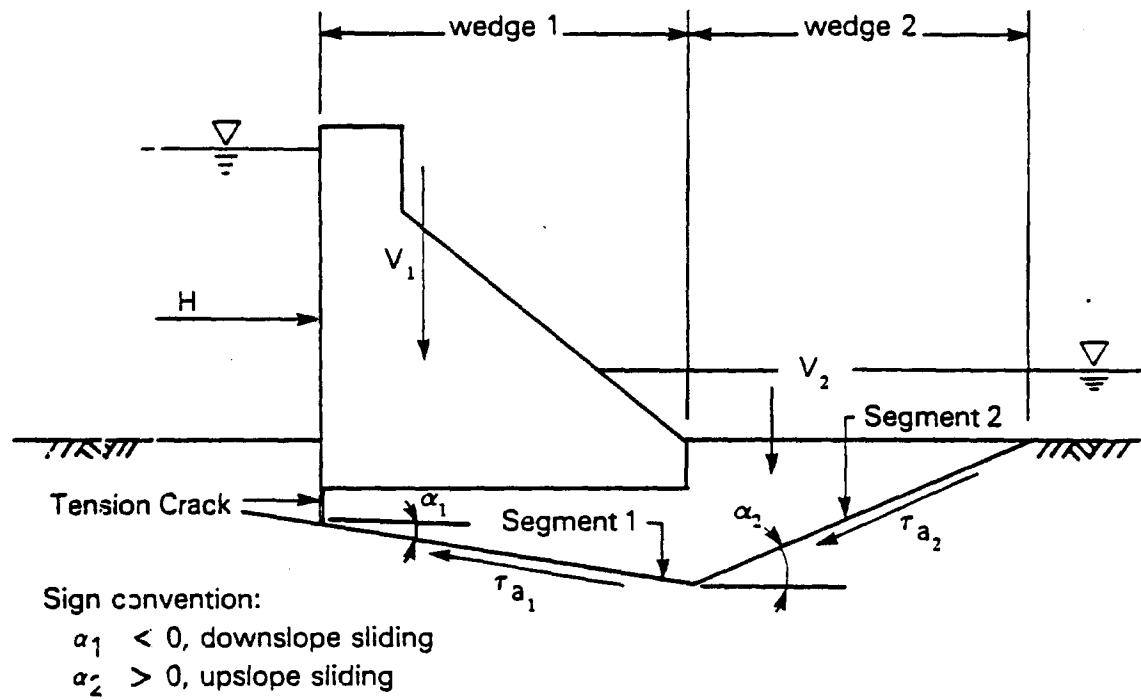
Figure 3-1. Single Plane Failure Mode

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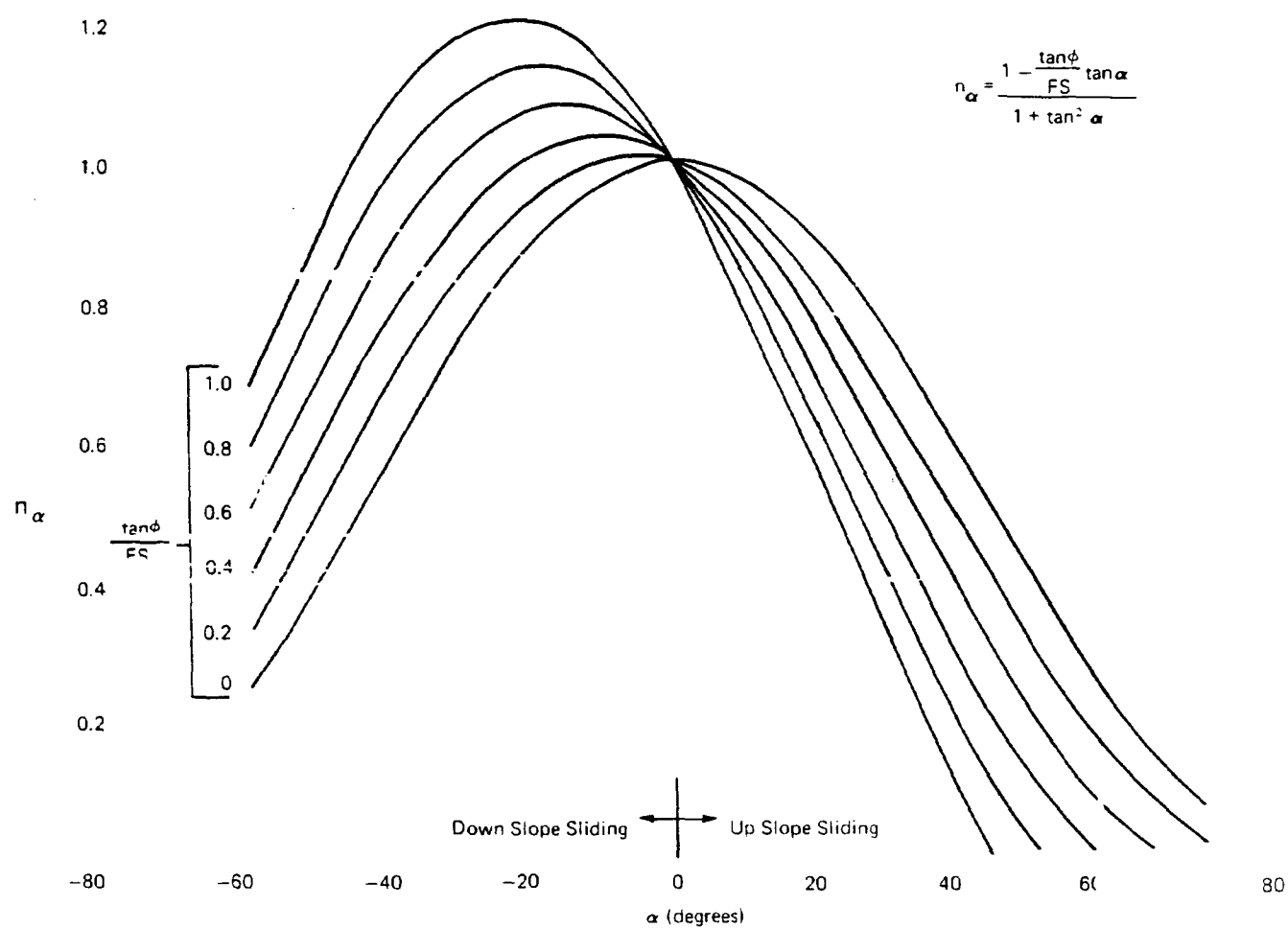Figure 3-2. Multiple Plane Failure Mode in the Simplist Form of Two Planes.

Figure 3-3. Plot of n_α and α for Values of $\tan\phi/FS$